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Larry B. Barrentine

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Preface

The intent of this manual is to provide background on the origins of the R&R methodology; help for those who work with devices that don't fit the usual approach; and ideas for measurement devices that require innovation to assess their performance under off-line, static conditions. The initial utilization of the R&R format is only briefly described; the reader can learn more about how to apply the procedures in 10 minutes of trying them than from several pages of instruction. R&R analysis is not a complete measurement assessment. However, it is often easier to start with the limited scope addressed by R&R. Once the R&R is assessed, the need to study further measurement issues can be evaluated. The ultimate objective is to determine how best to improve the control and performance of a process. The reader is assumed to be familiar with basic control charting methodology since assessment of statistical control of the measurement process is important.

The material is presented at multiple levels. One reader may concentrate on the "how-to" sections; another may wish to consider the derivations; a third may be interested in the interrelationships of measurement and process capability. Those who are interested in a more rigorous approach should review the appendices and references.

Since this manual was first published, several changes and options have evolved in the way R&R studies are analyzed. This revision updates the material in these areas:

1. The original format and procedures were based on a stand-alone form, the "GM long form." Current availability of computer support makes the use of a prepared form unnecessary. Spreadsheets are faster and easier, providing an easily filed and maintained record. The forms that were originally so useful now simply provide a sample format for spreadsheets. That format is referred to here as the "standard procedure."

2. The original R&R assessment criterion was made against the product tolerance. Today the assessment is typically calculated against the estimate of process variation. That measure is independent of the specifications and the criteria are more statistically defensible. Since these two assessment techniques answer very different questions, both are presented and the advantages of each are discussed. Preference is given to the assessment against the process variation.
3. The simplest analysis method uses ranges, and that classic approach remains preferred in this manual. Statistical packages are available, however, that can do the equivalent analysis using analysis of variance (ANOVA). Such packages provide a subtle option in the model used, involving the interaction of operator and sample. The application of ANOVA techniques is discussed with an example of typical output.
4. The interrelationship of the two criteria for R&R with process capability, represented by C_p , is shown to better illustrate the broad view needed to keep R&R priorities in perspective.

It is the author's contention that decision making on and evaluation of measurement systems should be done in the context of process capability. The particular criterion used for measurement capability is less important than the full context of measurement and process variation. Hopefully some of the points introduced here will help in process improvement efforts.

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1

Introduction


Just what is an “R&R” study, anyway? Well, it isn’t a study of “rest and relaxation!” R&R—repeatability and reproducibility—studies analyze the variation of measurements of a gage (repeatability) and the variation of measurements by operators (reproducibility). To understand why this is so important, recall that the goal of process control is reduction of variation in the process and ultimately the product. To address actual process variability, the variation due to the measurement system must be identified and separated from that of the process. Studies of measurement variation are a waste of time and money unless they lead to action to reduce process variation and improve process control. Since you can’t address something that cannot be measured precisely, the assessment of the gage becomes an early priority.

Before we can continue discussing R&R, we have to define “gage.” When asked to name a gage, people typically think of micrometers, pressure gages, temperature gages, and so on. However, the term “gage” actually refers to any device used for making measurements. In this manual, the terms “gage” and “device” are used interchangeably and refer to any device or equipment for making a measurement.

Every observation of a process contains both actual process variation and measurement variation (Figure 1.1). In the case of measurement systems, the sources are:

1. The gage
2. The operator
3. The variation within the sample

Gage variability can be broken into additional components, such as:

1. Calibration (Is the gage accurate?) 
2. Stability (Does the gage change over time?)

3. Repeatability (Is there variation of the gage when used by one operator in a brief time interval?)
4. Linearity (Is the gage more accurate at low values than at high values or vice versa?)

Variation within a sample is a part of process variation that is often mixed with measurement variation.

As noted, R&R studies assess reproducibility (operator variation) and repeatability (gage variation). Repeatability is the variation observed when an operator measures the same sample with the same gage several times. Reproducibility is the additional variation observed when several operators use the same gage to measure the same sample. The combination of both sources of variation is referred to as R&R (see Figure 1.2). Note that R&R does not address the *total* measurement system but is narrowly defined and is gage specific.

The exclusion of the other potential sources of measurement variation does not imply that calibration, stability, or linearity are unimportant; it is just that those sources are ordinarily less significant in their impact. For that reason, R&R are often studied and quantified first. In order to improve them, you must address the key measurement process variables via procedures, standards, training, and appropriate studies. We plan and execute R&R studies in a fashion designed to avoid confusion with sources of variation other than repeatability (gage) and reproducibility (operator). While this manual describes how to perform R&R studies, you cannot ignore the other sources of variation for long. In particular, the actual process variation is the ultimate subject to be addressed. Customers require both R&R studies and process capability. Process capability includes both process variation and measurement variation. Consequently, R&R studies should be accompanied or quickly followed by evaluations of calibration, variation within the sample, and any other relevant source of variability.

Variation within the sample being measured is often difficult to exclude from the R&R study. While not attributable to measurement, this source is *extremely important* and

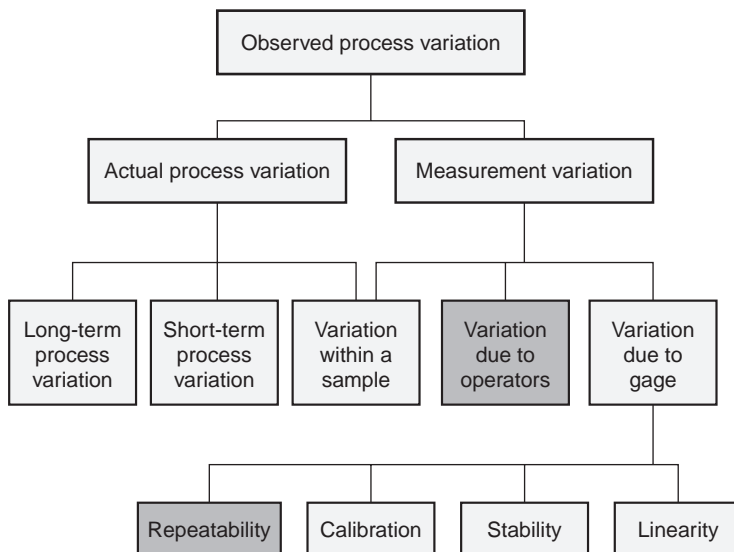


Figure 1.1 Possible sources of process variation.

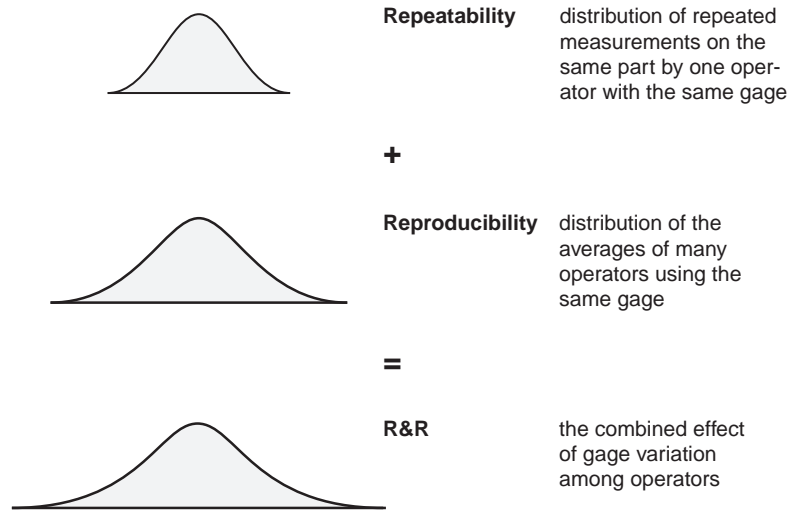


Figure 1.2 Repeatability, reproducibility, and R&R.

should always be pursued with diligence. It not only has relevance to understanding R&Rs but also provides vital information on how to gain process capability improvements.

A specific example of variation within the sample is apparent in measurements of surface texture by a profilometer. The test piece itself is sufficiently variable that if the measurement is made at a random position, the variation within the sample will inflate the estimate of repeatability. It is necessary to identify and measure this variability within the sample; this is not the role of an R&R study alone. Appendix E discusses this issue in more depth. The key point is to make certain that process variability within the sample does not intrude on an R&R study if it can be avoided. Determination of an unsatisfactory R&R should always lead to an evaluation of whether variation within the sample is part of the problem.

The impact of any environmental conditions needs to be evaluated. That is more appropriately addressed by designed experiments. In R&R studies an effort is made to block out such sources of variation.

It is necessary to introduce the mathematical version of Figure 1.2 since this relationship is used repeatedly. To add distributions, one must add the variances or σ^2 's of the distributions being added. If the distributions or spread due to repeatability and reproducibility can be characterized by their respective sigmas (σ_{repeat} and σ_{repro}), then combining these distributions as in Figure 1.2 results in the following distribution for R&R:

$$\sigma_{R\&R}^2 = \sigma_{repeat}^2 + \sigma_{repro}^2$$

The sigma for R&R ($\sigma_{R\&R}$) is the square root of this expression. This same Pythagorean relationship will be used in the appendices to relate the observed process variation to the measurement variation and the actual process variation; that is,

$$\sigma_{observed_process}^2 = \sigma_{actual_process}^2 + \sigma_{measurement}^2$$

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